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Long-term correlations in hourly wind speed records in Pernambuco, Brazil

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ABSTRACT

We study the statistical properties of hourly wind speed time series detected at four weather stations in the state of Pernambuco, Brazil, in the period 2008–2009. We find that the average and maximum hourly wind speeds deviate from a mutual linear relationship, and that they may be well explained individually by a Weibull distribution, however, with different shape parameter values. On the other hand, the long-term correlations of both of these observables obey the same power-law behavior, with two distinct scaling regimes. Our results agree with previous studies on wind speed series correlations in other regions of the world, which is suggestive of universal behavior.

1. Introduction

Long-term power-law correlations are characteristic of many phenomena including those in physiology [1,2], climatology [3–16], finance [17–19], ecology [20], and geophysics [21,22]. Indeed, over the last decade there has been a significant increase of the number of studies designed to analyze correlation properties of long series of observed and simulated climatic data on the global scale (see, e.g., Refs. [6–10] and references therein). Most of these studies are concerned with two principal aspects: (i) the universality of scaling exponents, and (ii) validation of existing global climate models. Various results on the correlation properties of surface temperature anomaly data indicate the existence of spatial variability of scaling exponents over land and ocean [6–8]; however, the geographic distribution of the correlation exponents and their dependence on the distance from the equator, the distance from the ocean, or elevation, all still remain open questions [6]. In fact, Govindan et al. [11] showed that seven state of the art global climate models fail to reproduce empirically observed scaling properties of long-term temperature records. To achieve better performance, diverse related phenomena, such as greenhouse gas concentration and volcanic forcing, have been included in some of the models [8,12].

As opposed to temperature, river discharge, and precipitation, which have been extensively studied in this context, there are only a few results on scaling properties of wind speed [3,13–16]. On the other hand, a better understanding of temporal and spatial variability of wind speed and direction is fundamental for modeling of various wind-related phenomena, including soil erosion [23], dune formation [24], air pollution [25], pollen and seed dispersal [26,27], evapotranspiration [28], and energy generation [29]. In this paper, we analyze long-term correlations in hourly wind speed temporal series registered in four sites in the state of Pernambuco, on the northeast of Brazil, using detrended fluctuation analysis (DFA) [30–34], which was designed [30] to quantify correlations in non-stationary signals.

In what follows, we first describe the data and present some exploratory statistical observations. Next, we present the DFA methodology used for the long-term correlation analysis, then we present the results with the accompanying discussion, and finally we draw the conclusions.

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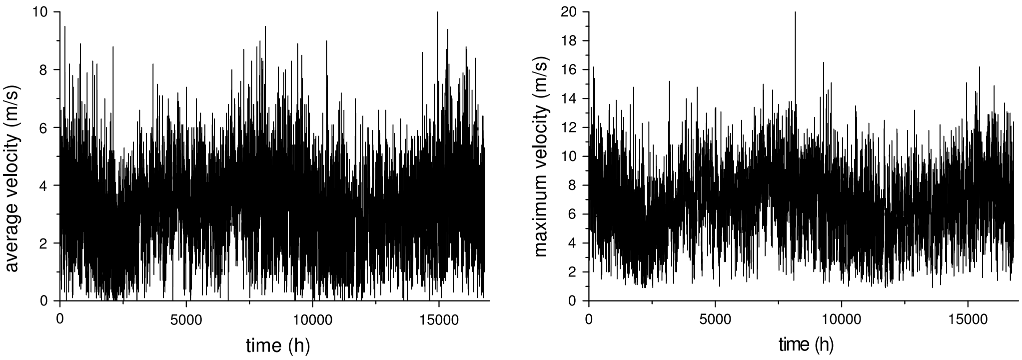

Fig. 1. The average and the maximum hourly wind speed data for the Arcoverde station.

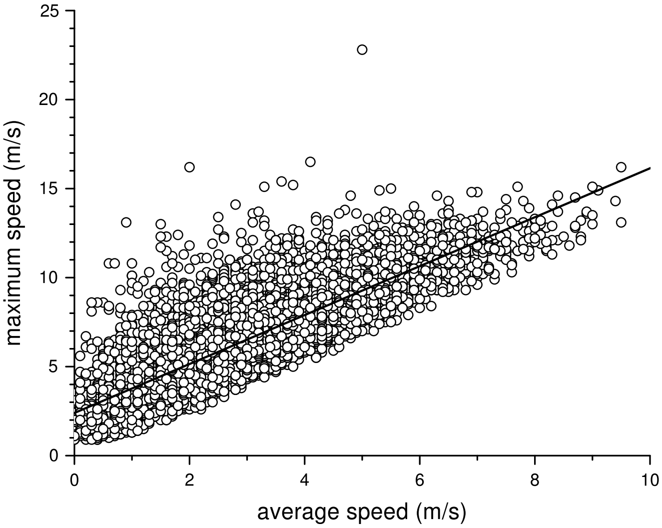

Fig. 2. The average versus maximum hourly wind speed data scatter plot for the Arcoverde station. The thick straight line represents a linear regression, with $R^2 = 0.833$.

Table 1
Latitude, longitude, and altitude of the selected meteorological stations.

Station	Latitude (degrees South)	Longitude (degrees West)	Altitude (m)
Arcoverde	8.4333	37.0833	680.70
Cabrobó	8.5036	39.3144	342.00
Garanhuns	8.9069	36.4972	822.00
Petrolina	9.3833	40.8000	370.46

2. Wind speed data

The data used in this work are provided by the Brazilian National Institute of Meteorology (Instituto Nacional de Meteorologia-INMET) [35]. We chose hourly wind speed time series detected by automatic meteorological stations in four sites which span the state of Pernambuco, Brazil, geographically and have hourly data available for the same full two-year period. Information on geographical location and altitude of these stations is given in Table 1. The observation period used is from January 1, 2008 to January 1, 2010, giving a total of roughly 17,000 hourly observation data points per station.

The raw time series for the average and maximum hourly wind speed presented in Fig. 1 for the station Arcoverde demonstrate a clear seasonal tendency, and the overall similarity between the two series is suggestive of a linear relationship between these two observables. In fact, one may observe by inspection of Fig. 1 that the average velocity data multiplied by a factor of two seem quite similar to the maximum velocity data, except for larger variance. A more quantitative relationship between the average velocity and the maximum velocity can be obtained by analyzing the cross-correlations between these two series, for example by applying the detrended cross-correlations analysis (DCCA) proposed in Ref. [36]. However, the scatter plot of these data presented in Fig. 2 reveals that the dispersion of the points around the regression line is asymmetric, with high-speed wind bursts occurring even for rather low average speed hourly intervals.

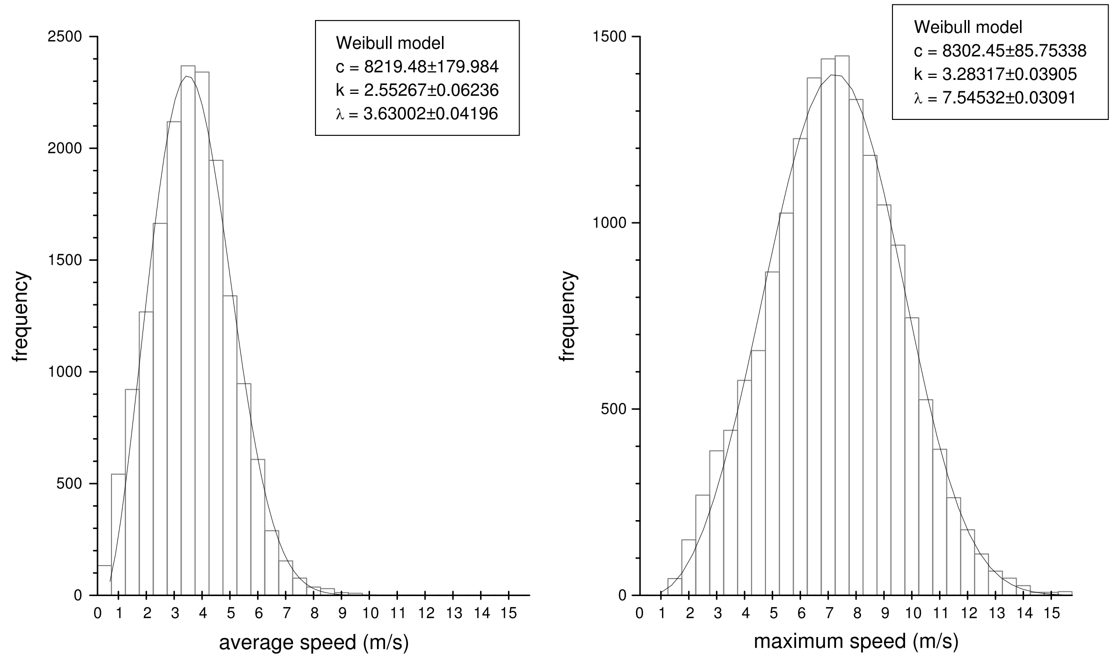

Fig. 3. The average and the maximum hourly wind speed histograms for the Arcoverde station. The continues lines represent regression to the Weibull model obtained using the Levenberg–Marquardt algorithm, where c is the multiplicative normalization constant reflecting the fact that the histograms are not normalized, k is the shape parameter, and λ is the scale parameter.

Table 2
Values of the shape parameter k and scale parameter λ of the Weibull distribution obtained through nonlinear regression of the wind speed histograms.

Station	Average speed		Maximum speed	
	k	λ	k	λ
Arcoverde	2.55267	3.63002	3.28317	7.54532
Cabrobó	2.24127	4.24000	2.50354	8.00892
Garanhuns	2.49402	3.41403	3.54476	7.38669
Petrolina	2.89692	3.82864	3.40184	7.89079

Therefore, in Fig. 3, we present the corresponding histograms with fits to a Weibull distribution, where nonlinear regression was implemented using the Levenberg–Marquardt algorithm. It is seen from Fig. 3 that both the average and the maximum wind speed distributions are well explained by a Weibull distribution (which is indeed the most widely used distribution for the description of wind speed data):

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, \tag{1}$$

however, with different shape and scale parameter values k and λ. To verify whether the wind velocity maxima are better explained using one of the extreme value distributions, we have also performed fits to Fréchet, Gumbel, and generalized extreme value distributions, and in all three cases we have obtained poor fitting.

The full set of shape and scale parameters, for the average and the maximum wind speed distributions at all four stations, is presented in Table 2, where it is seen that a similar conclusion holds for the Garanhuns and Petrolina stations, but in the case of Cabrobó the average and maximum speed are described by practically the same shape parameter.

3. Long-term correlations

3.1. Detrended fluctuation analysis

To quantify correlations in wind speed time series, we use detrended fluctuation analysis (DFA), introduced by Peng et al. [30]. This method is suitable for detecting long-term correlations in non-stationary signals [31,32], and it has been successfully applied to physiological signals [37,38], weather data [39], and financial time series [40,41]. The DFA procedure is described as follows. The original temporal series $x(i)$, $i = 1, \ldots, N$ is integrated to produce $y(k) = \sum_{i=1}^{k} [x(i) - \bar{x}]$, where $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x(i)$ is the average and k is an integer between 1 and N. Next, the integrated series $y(k)$ is divided into N_n non-overlapping segments of length n, and in each segment the linear (or higher-order polynomial) least-square fit (representing the local trend) is estimated. The integrated series $y(k)$ is then detrended by subtracting the local trend $y_i(k)$ (ordinates of the straight line or higher-order polynomial segments) from the data in each segment, and the detrended variance is calculated as

$$F_n^2 = \frac{1}{nN_n}\sum_{i=1}^{N_n}\sum_{k=(i-1)\,n+1}^{i:n}[y(k)-y_i(k)]^2\,. \quad (2)$$

Repeating this calculation for all box sizes provides the relationship between the root-mean-square fluctuation F_n and box size n, where typically F_n increases with n as a power law: $F_n \sim n^\alpha$. The scaling exponent α is obtained as the slope of the regression (least-squares line fitting) of $\log F_n$ versus $\log n$. The value of $\alpha = 0.5$ indicates the absence of correlation (white noise), $\alpha > 0.5$ indicates persistent long-term correlations, meaning that large (small) values are more likely to be followed by large (small) values, and $\alpha < 0.5$ indicates anti-persistent long-term correlations, meaning that large values are more likely to be followed by small values, and vice versa. The values $\alpha = 1$ and 1.5 correspond to $1/f$ noise and Brownian noise (integration of white noise), respectively. Use of higher polynomial order for detrending the series is sometimes necessary for capturing power-law behavior, where the choice of order $1, 2, 3, \ldots$ is denoted [33,34] DFA1, DFA2, DFA3, etc.

3.2. Multiple-range regression

Real-world data often display two or more ranges of power-law behavior, where corresponding intervals on the plot $\log F_n$ versus $\log n$ can be described well by lines of different slopes. In order to extract the exponents of two segments, one may assume the functional form

$$f(x) = (\alpha_1 x + \beta_1)\,\theta(x_0 - x) + (\alpha_2 x + \beta_2)\,\theta(x - x_0), \quad (3)$$

where

$$\theta(x) = \begin{cases} 0; & x < 0 \\ 1; & x \geq 0 \end{cases} \quad (4)$$

is the Heaviside theta function, $\{\alpha_1, \beta_1\}$ and $\{\alpha_2, \beta_2\}$ are the parameters of the two lines, respectively, and x_0 represents the crossover point where the lines intersect. In order to explain the rounding effect in the vicinity of the intersection, one may further replace the Heaviside step function $\theta\,(x - x_0)$ in (3) by its "smooth" equivalent $1/\left(1 + e^{-p(x-x_0)}\right)$, with an additional "rounding" parameter $p > 0$. One may therefore adopt the following functional form:

$$f(x) = \frac{\alpha_1 x + \beta_1}{1 + e^{p(x-x_0)}} + \frac{\alpha_2 x + \beta_2}{1 + e^{-p(x-x_0)}}, \quad (5)$$

and then perform least-squares fitting of the data using the Levenberg–Marquardt algorithm. It is straightforward to extend this procedure to situations with more than two segments.

3.3. Wind speed data power-law behavior

In order to make sure that daily periodicity does not affect the power-law behavior [32,33], in what follows we first calculate the 24 hourly averages, and use the difference between the observed values and the corresponding averages in the subsequent DFA. We further verify that there are no regions with high slope values which might stem from periodic effects, as demonstrated in Ref. [32], and we perform the modified DFA proposed in Ref. [33], where no localized peaks characteristic of periodic signal were observed. Finally, in order to make sure that the observed crossover behavior is not affected by any remnant periodicity of high frequency, we (a) shuffle the data on an hourly scale while keeping the daily data in the original order, and (b) we shuffle the daily data, while keeping the hourly data in order for each individual day. The results of these tests, displayed in Fig. 4, demonstrate that shuffling destroys correlations on the corresponding part of the scale in both cases, while behavior on the complementary part of the scale is preserved.

To verify whether DFA1 already captures scaling of the series, or if use of higher-order polynomials is necessary, in Fig. 5 we display the results of the DFA1, DFA2, and DFA3 analyses for the Arcoverde station mean wind speed data, together with the DFA1 analysis of the shuffled series. It is seen from Fig. 5 that DFA1 already captures well the behavior of the series, as the higher-order curves display identical behavior with two distinct linear regions, which is rather different from that of the shuffled data with a single slope of 0.5. Consequently, only DFA1 is used for data analysis in the rest of this paper.

Our main results are displayed in Fig. 6, where the DFA1 analysis results are shown for all four stations, for the average and the maximum hourly wind speed data, together with nonlinear regression to the five-parameter model given by (5), where the "rounding" parameter was set to $p = 10$, rather then being treated as an adjustable parameter. We have opted for fixing p because the two regions described by different scaling exponents contain different numbers of points, and treating p as a variable parameter produces an undesired effect of over-fitting with respect to the longer regime. It is seen from Fig. 6 that

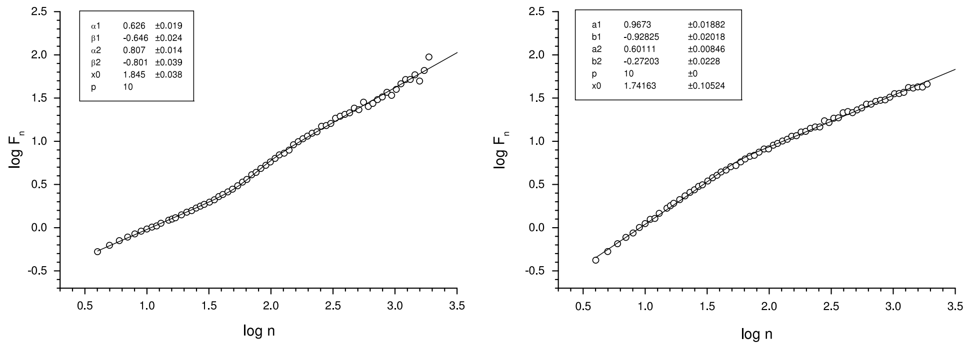

Fig. 4. The DFA1 analysis of the average wind speed data for the Arcoverde station, after (a) shuffling the data within each day, and (b) shuffling the days while preserving the hourly data order. The full lines represent regression to bilinear model (5). See Fig. 6 and Table 3 for comparison.

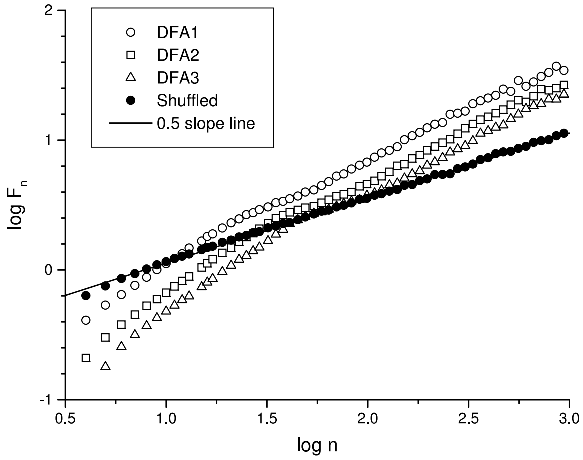

Fig. 5. Scaling behavior of the average hourly wind speed series for the Arcoverde station, using the DFA1, DFA2 and DFA3 analyses, together with results of the DFA1 analysis on the shuffled data.

Table 3
The values of the scaling exponents α_1 and α_2 and the crossover point x_0 of the wind speed time series (see the text for details).

Station	Average speed			Maximum speed		
	α_1	α_2	x_0	α_1	α_2	x_0
Arcoverde	1.06961	0.77440	1.36027	1.18182	0.78735	1.26208
Cabrobó	1.08468	0.87048	1.42138	1.17840	0.85553	1.41622
Garanhuns	1.09217	0.87832	1.25339	1.22060	0.84666	1.35157
Petrolina	1.07234	0.88818	1.36508	1.03593	0.85257	1.39796

the average and the maximum hourly wind speed series data display identical behavior for all four stations, and can hardly be distinguished from each other. Excellent agreement of the calculated data points with the five-parameter model (5) is observed in all cases, including the rounding of the crossover between the two scaling regimes. Crossover between different regimes has been observed in other phenomena [42–44]. The values of the slopes α_1 and α_2 and the crossover point x_0 are given in Table 3.

The crossover point between the two scaling regions is found to be in the range $1.25 \leq x_0 \leq 1.42$, which corresponds to the range between $10^{1.25} \sim 18$ h and $10^{1.42} \sim 26$ h. For temporal scales longer than x_0, the scaling exponents are in all cases greater than 0.5, which confirms the existence of power-law long-term correlations. For scales shorter than x_0, the fluctuations become closer to $1/f$ noise (the value of the scaling exponent for all stations is close to 1.1). Similar results are obtained for wind speed data in Denmark, the USA, and Turkey [3,13,15], which indicates possible universal global behavior of wind speed fluctuations.

Our results suggest that the long-term correlations exhibit the same behavior across the state of Pernambuco, where Arcoverde and Garanhuns stations are relatively close to the seacoast, while Cabrobó and Petrolina are deep in the interior with semi-arid climate, while classical statistical analysis of the wind speed distributions does not offer systematic

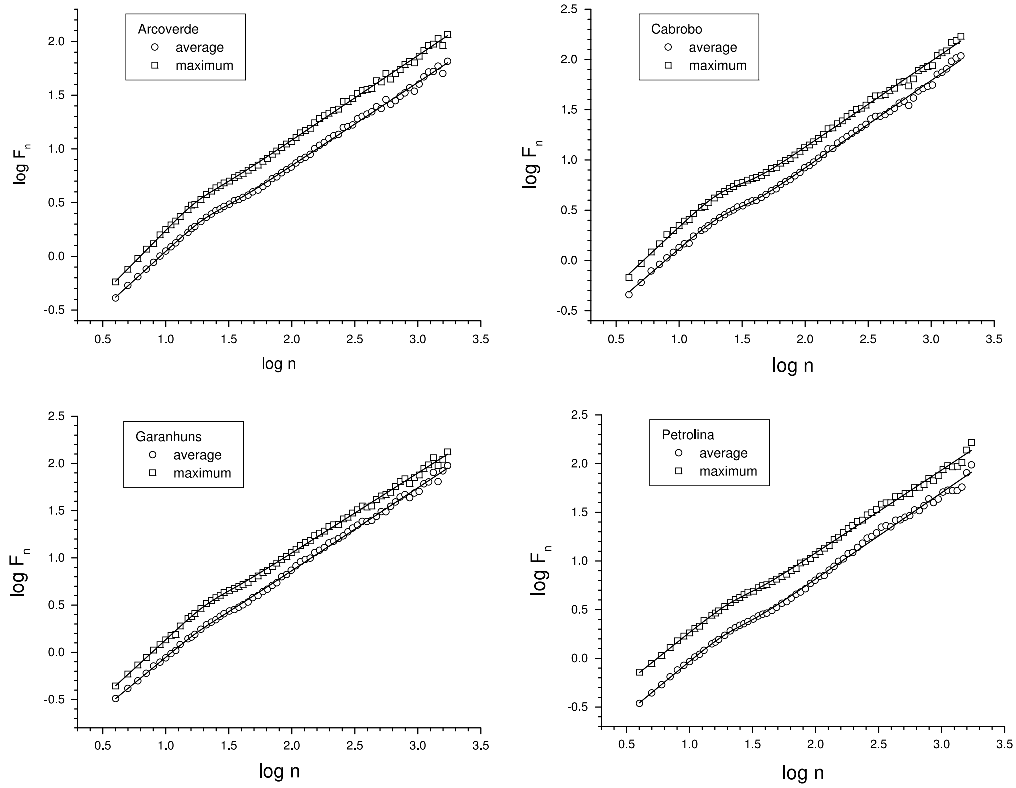

Fig. 6. The DFA1 analysis results for the average and the maximum hourly wind speed data for all four stations. The continuous lines represent regression to bilinear model (5) (see the text for more details).

conclusions. Nevertheless, in order to make conclusive statements about the geographic distribution of the scaling exponents, it is necessary to analyze data from more weather stations in different spatial locations, as well as gridded data generated by global climate models.

4. Conclusion

Our results contribute to a better understanding of wind speed dynamics, which is crucial for developing theoretical and computational models for various wind-related phenomena, by establishing equivalence of the average and maximum hourly wind speed distributions across the state of Pernambuco, Brazil. Classical statistical analysis of wind speed distributions on the other hand offers somewhat erratic results, where the average and maximum wind speed are in some cases found to be equivalent, and in other cases rather different from each other, with no clear indication as to the cause of these similarities and/or differences in terms of geographical position, altitude, or climate. Our findings should prove useful for validation of global climate models, since a valid model should explain empirically detected scaling properties in observed data. Current results should be compared with model-generated data for spatial locations (longitude and latitude) that correspond to the locations of meteorological stations used in this work. A comparative analysis of this kind was performed with precipitation data in Spain [45].

In conclusion, the hourly wind speed temporal series registered in the state of Pernambuco, Brazil, display long-term correlations with two distinct scaling regions. For temporal scales longer than one day, the scaling exponent is greater than 0.5, meaning that the original series are persistent; for shorter time scales, the scaling exponent is close to unity, which is characteristic of $1/f$ noise. Our results are in good agreement with results obtained for other locations, indicating the possibility of universality with respect of geographic parameters. To test this hypothesis, it is necessary to analyze wind speed data at different geographic spatial locations.

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